Decision Theory

1.3 Decisions Under Risk

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What should you do in a choice situation where you can assign probabilities to the outcomes of each of your potential actions?

Ex. Pharmaceutical Company.

You are the CEO of a pharmaceutical company that has developed an insomnia drug which just received FDA approval. However, there is a 10% probability that the drug has bad side effects that were not detected in the FDA trials, and you are considering whether to run an additional \$1M test to find out if the drug has these effects (should the test reveal the bad side effects, you will not market the drug as planned). If you market the drug and there are no problems, then you stand to make \$5M in sales. But if you market the drug without further testing and it has problems, then you stand to lose \$15M. Do you run the test?

For the time being, we assume that acts and states are independent, and probabilities can be assigned to each state.

Def 1.3.1. A *decision model* $\mathcal{M} = \langle \mathcal{A}, \Omega, \mathcal{P}, \mathcal{O}, g \rangle$ consists of a set of actions $\mathcal{A}$, a set of states Ω, a *probability measure* $\mathcal{P} : 2^\Omega \to \mathbb{R}[0, 1]$, a set of outcomes $\mathcal{O}$, and a function $g : \mathcal{A} \times \Omega \to \mathcal{O}$ that maps each action $a \in \mathcal{A}$ and state $s \in \Omega$ to an outcome $o \in \mathcal{O}$.

2^Ω is the *power set* of Ω (the set of all sets of states in Ω).

Assuming that your utility is linear in dollars, we can model Pharmaceutical Company as follows:

	$[\frac{1}{10}]$ bad effects	$[\frac{9}{10}]$ no bad effects
test	-1	4
no test	-15	5

Should you run the additional test or not?

Def 1.3.2. The *expected utility* of $a \in \mathcal{A}$ is $\sum_{s \in \Omega} Pr(s) \times u(g(a, s))$.

This will be designated by $EU(a)$.

Thesis 1.3.1 (Principle of Maximizing Expected Utility). Rationality forbids any act $a_1 \in \mathcal{A}$ such that $EU(a_1) < EU(a_2)$ for some alternative act $a_2 \in \mathcal{A}$.

In other words, maximize EU.

The Principle of Maximizing EU presupposes an interval utility function.

	$[\frac{1}{10}]$	$[\frac{9}{10}]$
	bad effects	no bad effects
test	-1	4
no test	-15	5

$$EU(\text{test}) = \frac{1}{10} \times -1 + \frac{9}{10} \times 4 = 3.5.$$

	$[\frac{1}{10}]$ bad effects	$[\frac{9}{10}]$ no bad effects
test	-1	4
no test	-15	5

$$EU(\text{no test}) = \frac{1}{10} \times -15 + \frac{9}{10} \times 5 = 3.$$

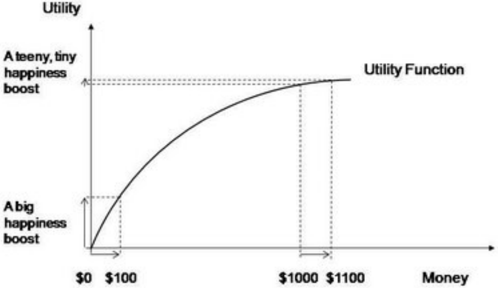
	$[\frac{1}{10}]$ bad effects	$[\frac{9}{10}]$ no bad effects
test	-1	4
no test	-15	5

$$EU(\text{test}) = 3.5 > EU(\text{no test}) = 3.$$

	$[\frac{1}{10}]$ bad effects	$[\frac{9}{10}]$ no bad effects
test	-1	4

What are we maximizing?

- Expected Monetary Value (*EMV*)
- Expected Value (*EV*)
- Expected Utility (*EU*)



These are not equivalent, though at times we'll talk as if they are. We'll generally focus on maximizing *EU*, and at times just assume that $EMV = EU$.

Why should we maximize *EU*?

- Law of Large Numbers Argument
    - Over the long run, those who maximize expected utility will "almost certainly" be better off.
    - As the number of trials performed grows, the probability that the average outcome differs from the expected outcome becomes arbitrarily small.
- Keynes: "in the long run we are wall dead," so what reason could we have for taking long run convergence into account in making a single decision?
- *Gambler's Ruin:* Over the course of a sufficiently large number of trials, we're sure to encounter a sequence of outcomes that we cannot afford. (Consider the $1 coin toss scenario.)

Why should we maximize EU?

- Axiomatic Approach
  - We can derive the expected utility principle from axioms that we find broadly acceptable.
  - Direct vs. Indirect
  - Direct Axiomatization
    - **Ax1.** If all outcomes of an act have a utility u, then the utility of the act is u.
    - **Ax2.** If one act dominates another, then former has a greater utility; if two acts weakly dominate one another, then they have the same utility.
    - **Ax3.** Every decision problem can be transformed into a decision problem with equally probable states in which the utility of all acts is preserved.
    - **Ax4.** If two outcomes are equally probable, and if the better outcome is made slightly worse, then this can be compensated by adding some amount of utility to the other outcome, such that the overall utility of the act is preserved.
  - Ax1-Ax4 entail that the utility of an act equals its expected utility and *vice versa*. The most attractive act is the act with the highest expected utility.

Ex. Monte Hall.

You are a contestant in a game show hosted by Monte Hall. There are three doors 1, 2, and 3. A car has been placed randomly behind one of these doors. Behind the other two doors are goats. Monte explains the rules: "First you will pick a door. Then I will open one of the doors that you did not pick. I know what is behind the doors so I will always reveal a goat. After I show you the goat, you will then have the option of switching your initial choice to the other door that I did not open." You initially pick A and Monte then opens C to reveal a goat. Do you stick with A or switch to B?

	[?] car behind A	[?] car behind B
stick with A	win car	win goat
switch to B	win goat	win car

	$[\frac{1}{3}]$ car behind A	$[\frac{2}{3}]$ car behind B
stick with A	win car	win goat
switch to B	win goat	win car

	$[\frac{1}{3}]$ car behind A	$[\frac{2}{3}]$ car behind B
stick with A	15	3
switch to B	3	15

$$EU(\text{stick with A}) = \frac{1}{3} \times 15 + \frac{2}{3} \times 3 = 7.$$

	$[\frac{1}{3}]$ car behind A	$[\frac{2}{3}]$ car behind B
stick with A	15	3
switch to B	3	15

$$EU(\text{switch to B}) = \frac{1}{3} \times 3 + \frac{2}{3} \times 15 = 11.$$

	$[\frac{1}{3}]$ car behind A	$[\frac{2}{3}]$ car behind B
stick with A	15	3
switch to B	3	15

$$EU(\text{switch to B}) = 11 > EU(\text{stick with A}) = 7.$$

	$[\frac{1}{3}]$ car behind A	$[\frac{2}{3}]$ car behind B
switch to B	win goat	win car

Ex. Mereen Hardwood Company

Mereen Hardwoods is considering expanding their mill and shipping operation at an inland port in Virginia to increase their capacity for filling orders from overseas buyers. They will either expand their operation by half, double its size, or maintain the status quo. Their market research tells them that there is a 20% chance that demand for American hardwoods will increase considerably in coming years and a 10% chance that it will fall. Otherwise, it will remain the same. If they double the operation, they will be well positioned to take advantage of increased demand and expect to realize a net profit of \$2.5M. However, if they double their size and the market remains the same, they will only realize a net profit of \$0.5M. If demand falls, they expect a net loss of \$0.5M. If they expand the operation by half, in good market conditions they'll see a net profit of \$1.5M. If demand holds steady, \$1M. And, if demand falls, \$0. Finally, if they maintain their current operation, whether demand remains the same or grows, they will realize a net profit of \$1M. If it falls, their net profit will be \$0.5M. What should Mereen Hardwood do? (Assume EU=EMV.)

What's the formal model $\mathcal{M}$ of Mereen Hardwood?

- $\mathcal{A} = \{\text{big expansion}, \text{small expansion}, \text{no expansion}\}$
- $\Omega = \{\text{increased demand}, \text{steady demand}, \text{falling demand}\}$
- $\mathcal{P} = Pr(\text{increased demand}) = .2$
  $Pr(\text{steady demand}) = .7$
  $Pr(\text{falling demand}) = .1$
- $\mathcal{O} = \{\$2.5\text{M}, \$1.5\text{M}, \$1\text{M}, \$0.5\text{M}, \$0, \text{-}\$0.5\text{M}\}$
- $g(\text{big expansion}, \text{increased demand}) = \$2.5\text{M}$
  $g(\text{big expansion}, \text{steady demand}) = \$0.5\text{M}$
  $g(\text{big expansion}, \text{falling demand}) = \text{-}\$0.5$
  $g(\text{small exmpansion}, \text{increasing demand}) = \$1.5$
  $g(\text{small expansion}, \text{steady demand}) = \$1\text{M}$
  And so forth.

Whats the decision matrix for Mereen Hardwood?

	[0.2] increase demand	[0.7] steady demand	[0.1] falling demand
big expansion	2.5	0.5	-0.5
small expansion	1.5	1	0
no expansion	1	1	0.5

Whats the decision matrix for Mereen Hardwood?

	[0.2] increase demand	[0.7] steady demand	[0.1] falling demand
big expansion	2.5	0.5	-0.5
small expansion	1.5	1	0
no expansion	1	1	0.5

$$EU(\text{big expansion}) = (0.2)(2.5) + (0.7)(0.5) + (0.1)(-0.5) = 0.8$$
$$EU(\text{small expansion}) = (0.2)(1.5) + (0.7)(1) + (0.1)(0) = 1$$
$$EU(\text{no expansion}) = (0.2)(1) + (0.7)(1) + (0.1)(.5) = .95$$

$$EU(\text{small expansion}) > EU(\text{no expansion}) > EU(\text{big expansion})$$

$$\text{small expansion} \succ \text{no expansion} \succ \text{big expansion}$$

Note that the Principle of Maximizing Expected Utility brings together two different ingredients—*viz.*, probability and utility. For a proper treatment of decisions under risk, then, we need a theory of probability and a theory of utility.

These are what we'll be looking at in more detail in the next few classes.