Game Theory

2.2 Nonzero Sum Games

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Things are no longer so clean once we move on to nonzero sum games. The equilibria of these games often lack the nice properties of equilibria of strictly competitive games.

Ex. Stag Hunt.

You and an acquaintance have gone hunting and you must each decide whether to pursue a large stag or to pursue a small hare. Alone, you can each capture a hare. However, a stag requires two people to capture. If you cooperate and both hunt stag, then you will end up with much meat. But if you hunt stag while your acquaintance hunts hare, then you return home empty-handed. Do you hunt stag or hare?

	hunt stag	hunt hare
hunt stag	2,2	0,1
hunt hare	1,0	1,1

Nash equilibria?

	hunt stag	hunt hare
hunt stag	2,2	0,1
hunt hare	1,0	1,1

$max_{a_1 \in \mathcal{A}_1}(min(a_1)) = 1$

$max_{a_2 \in \mathcal{A}_2}(min(a_2)) = 1$

Moral. Playing maximinimizers will result in the suboptimal Nash equilibrium $\langle \text{hunt hare}, \text{hunt hare} \rangle$. Individual rationality alone cannot lead to the optimal Nash equilibrium $\langle \text{hunt stag}, \text{hunt stag} \rangle$. It is useful to develop incentive mechanisms for cooperation.

Moral. Not all Nash equilibria of a nonzero sum game have the same utilities. We cannot always speak of the *value* of a game.

Pareto Optimality. In both economics and decision theory, a Pareto Optimal/Efficient outcome is one such that no one can be made better off without making someone else worse off. It's often taken as a kind of baseline for rational outcomes of games.

What's the Pareto Optimal outcome for Stag Hunt?

Ex. Bach or Stravinsky? (Osborne and Rubinstein)

Two people wish to go out together to a concert of music by either Bach or Stravinsky. Their main concern is to go out together, but one person prefers Bach and the other person prefers Stravinsky. Suppose that have expressed their wishes to one another, but have not been able to coordinate an outcome (perhaps someone's phone battery has died). What should they do?

This is a variant on the classic Battle of the Sexes (Luce and Raiffa), sometimes also called the Clash of Wills.

	Bach	Stravinsky
Bach	3,2	1,1
Stravinsky	0,0	2,3

Nash equilibria?

	Bach	Stravinsky
Bach	3,2	1,1
Stravinsky	0,0	2,3

$max_{a_1 \in \mathcal{A}_1}(min(a_1)) = 1$

$max_{a_2 \in \mathcal{A}_2}(min(a_2)) = 1$

Moral. Playing maximinimizers will result in the non-equilibrium $\langle \text{Bach}, \text{Stravinsky} \rangle$. Individual rationality leads to an unstable solution.

Alternative. What if each player, assuming the other is rational, thinks that the other will make the decision that she would make in the other's shoes? Will they coordinate?

Ex. Hawk & Dove (Osborne and Rubinstein).

"Two animals are fighting over some prey. Each can behave like a dove or like a hawk. The best outcome for each animal is that in which it acts like a hawk while the other acts like a dove; the worst outcome is that in which both animals act like hawks. Each animal prefers to be hawkish if its opponent is dovish and dovish if its opponent is hawkish."

This is a variant on the classic Chicken.

	dove	hawk
dove	2,2	1,3
hawk	3,1	0,0

Nash equilibria?

	dove	hawk
dove	2,2	1,3
hawk	3,1	0,0

$max_{a_1 \in \mathcal{A}_1}(min(a_1)) = 1$

$max_{a_2 \in \mathcal{A}_2}(min(a_2)) = 1$

Moral. Playing maximinimizers will result in the non-equilibrium $\langle \text{Dove}, \text{Dove} \rangle$. Again, individual rationality leads to an unstable solution.

Ex. Prisoner's Dilemma (Raiffa, Flood and Dresher, Tucker).

Two individuals are know to have committed minor offenses but they are also suspected of robbing a bank together. They are arrested and placed in separate interrogation rooms. If they both confess to the robbery, then they will be sentenced to 5 years in prison. If only one confesses, then he will be set free but used as a character witness against the other individual who will receive a sentence of 10 years. If neither confesses, then they will both be convicted of the minor offenses and sentenced to only 1 year in prison.

	confess	do not confess
confess	-5,-5	0,-10
do not confess	-10,0	-1,-1

Nash equilibria?

	confess	do not confess
confess	-5,-5	0,-10
do not confess	-10,0	-1,-1

Moral. The only Nash equilibrium $\langle \text{confess}, \text{confess} \rangle$ is suboptimal. Individual rationality can lead to outcomes that are not best for the group.

Real-life PDs:

- Arms races
- Vampire bats
- Doping in sports
- Drug addiction
- Carbon dioxide emissions

Lewis (and others): Prisoner's Dilemma is a Newcomb Problem.

To confess, you take a transparent box containing \$1000 (like taking the Queen's shilling).

If you do not confess, then the other prisoner receives \$1M.

	confess	do not confess
confess	\$1000,\$1000	\$1M + \$1000,\$0
do not confess	\$0,\$1M+\$1000	\$1M,\$1M

Suppose that whether a prisoner receives \$1M is causally independent of what they do, but that each prisoner believes for good reason that the other prisoner will act like them. Should they confess?

Ex. Traveller's Dilemma (Basu).

An airline loses your suitcase and the suitcase of your doppelgänger that has the exact same contents. An airline manager separates you and your doppelgänger and asks you both to estimate the value of your lost luggage at no less than \$2 and no more than \$100 which is the maximum that the airline will reimburse you. If you both write down the same number, then the manager will treat this as the true value of your luggage and reimburse you both this amount. But if you write down different numbers, then the manager will treat the lower number as the true value. Moreover, whichever one of you wrote down the lower number will be awarded \$2 extra for your honesty, and whichever one of you wrote down the higher number will have \$2 deducted from your payout. What number do you write down?

Partial game matrix:

	\$2	\$3	\$4	\$5
\$2	\$2,\$2	\$4,\$0	\$4,\$0	\$4,\$0
\$3	\$0,\$4	\$3,\$3	\$5,\$1	\$5,\$1
\$4	\$0,\$4	\$1,\$5	\$4,\$4	\$6,\$2
\$5	\$0,\$4	\$1,\$5	\$2,\$6	\$5,\$5

Nash equilibria?

Partial game matrix:

	\$2	\$3	\$4	\$5
\$2	\$2,\$2	\$4,\$0	\$4,\$0	\$4,\$0
\$3	\$0,\$4	\$3,\$3	\$5,\$1	\$5,\$1
\$4	\$0,\$4	\$1,\$5	\$4,\$4	\$6,\$2
\$5	\$0,\$4	\$1,\$5	\$2,\$6	\$5,\$5

The only Nash equilibrium is $\langle \$2, \$2 \rangle$

Partial game matrix:

	\$2	\$3	\$4	\$5
\$2	\$2,\$2	\$4,\$0	\$4,\$0	\$4,\$0
\$3	\$0,\$4	\$3,\$3	\$5,\$1	\$5,\$1
\$4	\$0,\$4	\$1,\$5	\$4,\$4	\$6,\$2
\$5	\$0,\$4	\$1,\$5	\$2,\$6	\$5,\$5

Note that the \$2-\$3 region is a Prisoner's Dilemma.